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ADVECTION-DIFFUSION MODEL WITH TIME DEPENDENT FOR AIR POLLUTANTS DISTRIBUTION IN UNSTABLE ATMOSPHERIC CONDITION

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ABSTRACT

Air pollution levels are quite high in urban areas. They are emitted from various sources and have an impact on humans and the environment. There are some physical processes that occur when pollutants disperse in the atmosphere. The main processes are advection and diffusion. Therefore, a two-dimensional mathematical model is presented to study the dispersion of air pollution under the effect of mesoscale wind as an effect of urban heat islands. This model is solved by using the implicit Crank-Nicolson finite difference scheme under stability-dependent meteorological parameters involved in large scale wind, mesoscale wind and eddy diffusivity. The main goal of this research is to analyze air pollution distribution using the advection-diffusion model. The results of this model have been analyzed for the dispersion of air pollutants in an urban area in the downwind and vertical direction for unstable atmospheric conditions.

Key words : Advection, Diffusion, Mesoscale Wind, Pollutant Dispersion

INTRODUCTION

Background

The aim of this study is to analyze pollutants dispersion under the effect of mesoscale wind using advection-diffusion equation. Strength of wind determines the movement of pollutants. The wind speed determines the depth of how much air pollutants are initially mixed and irregularities speed and wind direction determines the spread of contaminants when carried in the wind direction. These factors determine an area to be contaminated and how quickly pollutants thinned due to mixing with ambient air after leaving the source. It can either pollute the atmosphere of other countries or pollute the atmosphere of other settlements around the site. High wind speeds and variation of wind directions cause pollutants to move far away from source thereby reducing concentration of pollutants near source (Isikwue *et.al,* 2011).

Agarwal and Tandon (2009) studied modeling of the urban heat island in the form of mesoscale wind and of its effect on air pollution dispersal and found the model for two-dimensional advection diffusion equation at steady state condition. In their study, they formulated a mathematical model which was used to study the dispersion of pollutants under the effect of urban heat island (UHI). Wind plays an important role on the dispersion of pollutants, it transports pollutants horizontally by a large-scale wind and mesoscale wind transports them in horizontal as well as in vertical direction; the mesoscale wind represents a local wind produced by UHI effect. A study by Narayanachari *et.al*., (2014) revealed that the impact of urban heat islands produce their own mesoscale wind and it prevents the dispersion of pollutants that would result in an increase in concentration of pollution in the atmosphere and Lakzhminarayanachari *et.al.,* (2013) also revealed that the urban heat islands increase the amount of haze pollutants contamination and also help pollutants to circulate upwardly resulting in pollution becoming more severe. In addition, Pandurangappa *et.al.,* (2012) stated that the mesoscale wind reduces the concentration of primary and secondary pollutants in the upwind side center of heat island and enhances the concentration in the downwind side of the center of the heat island, and as the gravitational settling velocity increases, the concentration of secondary pollutants decreases.

Quantification of the relationships between emissions and concentrations involves two steps: firstly by identifying what the most important atmospheric processes are and secondly representing these mathematically in a model (Bell and Treshow, 2002). Mathematical models are useful to study how pollutants behave when there are new sources of air pollution or changes in the amount of pollutants emitted into the air from the presence of emission sources and help in analyzing such behaviors (Awasthi, Khare and Gargava, 2006). Mathematical formulations of transport and dispersion are developed to identify the parameters of interest. Air quality models have become integrated tools in environmental monitoring, management and assessment of air pollution (Fenger and Tjell, 2009). The perfect air pollutant concentration model would allow us to predict the concentrations that would result from any specified set of pollutants emissions, for any specified meteorological conditions, at any location, for any time period, with total confidence in our prediction (de Nevers, 2000). A study by Arystanbekova (2004) stated that simulation of air pollution is useful in providing information about the spread of pollutants in an area, the scale, level of pollution and estimation.

METHODOLOGY

There are some physical processes that occur when pollutants disperse in the atmosphere. The main processes are advection and diffusion; advection is the air movement which transports pollutants by wind as a bulk motion, while diffusion is the particle movement from high concentration to low concentration due to collision between the particles and the movement will stop when it reaches equilibrium. Diffusion is caused by turbulent; it is the irregular air movement where by the wind constantly varies in speed and direction.

The mathematical formulation of the air pollution dispersion is based on the conservation of mass equation which describes advection, turbulent diffusion, chemical reaction, etc (Agarwal and Tandon, 2009). The advection-diffusion equation is stated as:

$$U\frac{\partial C}{\partial x}+V\frac{\partial C}{\partial y}+W\frac{\partial C}{\partial z}=\frac{\partial}{\partial x}\left(K_x\frac{\partial C}{\partial x}\right)+\frac{\partial}{\partial y}\left(K_y\frac{\partial C}{\partial y}\right)+\frac{\partial}{\partial z}\left(K_z\frac{\partial C}{\partial z}\right)+R, \qquad (1)$$

where C is the air pollutant concentration at any location (x, y, z); R is the removal / reaction term; u, v and w are the wind components in downwind (x), crosswind (y) and vertical (z) directions respectively; K_x, K_y and K_z are eddy diffusivity coefficients in x, y and z direction respectively.

The equation (1) describes physical processes of a substance whit advection and diffusion in it. Advection carries pollutants in the direction of wind and diffusion carries pollutants from a region of high concentration to a region of low concentration by random movements. These random movements are caused by turbulence and convection. There is removal mechanism of pollutants in the atmosphere by precipitation or gravitational settling.

In this study, the mathematical formulation used is an advection diffusion model with time dependence which is stated as:

$$\frac{\partial C}{\partial t}+U\frac{\partial C}{\partial x}+V\frac{\partial C}{\partial y}+W\frac{\partial C}{\partial z}=\frac{\partial}{\partial x}\left(K_x\frac{\partial C}{\partial x}\right)+\frac{\partial}{\partial y}\left(K_y\frac{\partial C}{\partial y}\right)+\frac{\partial}{\partial z}\left(K_z\frac{\partial C}{\partial z}\right)+R, \qquad (2)$$

where the variable of interest is the concentration of pollutants (C) and the parameters involved in the advection process are wind components which are represented by U, V and W in each direction x, y and z respectively. While in the diffusion process, the parameters are eddy diffusivities which are represented by K_x, K_y and K_z in each direction and for the removal mechanism, it is represented by R.

Some studies of pollutant dispersion in the atmosphere have only been on simulation, so in this study, we do visualization as well with the model which takes into account large scale wind, low wind and eddy diffusivity profile with the following assumptions.

i) Urban terrain is homogeneity; the mean concentration of pollutants is constant along crosswind direction. So, there is no y −dependence.
ii) The horizontal advection by the wind dominates over horizontal diffusion. Hence the concentration in the diffusion term is negligible.
iii) Considering for the removal term with a constant rate on the concentration.

Under those assumptions equation (2) becomes :

$$\frac{\partial C}{\partial t} + U\frac{\partial C}{\partial x} + W\frac{\partial C}{\partial z} = \frac{\partial}{\partial z}\left(K_z\frac{\partial C}{\partial z}\right) - \lambda C \qquad (3)$$

where $C = C(x, z, t)$ is the concentration of pollutants, U is the wind speed in $x-$direction, W is the wind speed in $z-$direction, K_z is the turbulent eddy diffusivity in $z-$direction and λ is a first order constant depletion parameter that defines the fractional loss of pollutants per unit time through various wet deposition processes existing in the atmosphere. It is known that in urban area the heat causes the vertical flow of air with rising of air at the center of the urban area, hence it can be called as heat island (Narayanachari *et.al.*, 2014). This rising of air forms an air circulation at larger heights. This is called mesoscale circulation. So, for wind velocity profile, it is expressed by large wind u and mesoscale wind u_e in horizontal direction and mesoscale wind w_e in vertical direction. The equation (3) can be rewritten as:

$$\frac{\partial C}{\partial t} + (u + u_e)\frac{\partial C}{\partial x} + (w_e)\frac{\partial C}{\partial z} = \frac{\partial}{\partial z}\left(K_z\frac{\partial C}{\partial z}\right) - \lambda C. \quad (4)$$

The large-scale wind u and the vertical diffusivity K_z as a function of vertical height z, as suggested by Lin and Hildemann (1995) is:

$$u = u_r\left(\frac{z}{z_r}\right)^{\alpha},$$

$$K_z = K_r\left(\frac{z}{z_r}\right)^{\beta}$$

where u_r and K_r are the measured wind speed and vertical diffusivity at reference height z_r and α, β are the constants which depends on the atmospheric stability and surface roughness. Considering mesoscale circulation, Dilley and Yen (1971) suggested

$$u_e = -ax\left(\frac{z}{z_r}\right)^{\alpha}, \quad w_e = \frac{az}{(\alpha+1)}\left(\frac{z}{z_r}\right)^{\alpha}.$$

The equation (4) is too complicated to be solved analytically because there are wind speed and eddy diffusivity profiles which are considered in the model. Therefore, it would be solved by the implicit Crank-Nicolson finite difference scheme,

$$\left[\frac{\partial C}{\partial t}\right]_{i,j}^{n+\frac{1}{2}} = \frac{1}{2}\left[\frac{\partial C}{\partial t}\bigg|_{i,j}^{n+1} + \frac{\partial C}{\partial t}\bigg|_{i,j}^{n}\right] \qquad (5)$$

where (i, j) indicate position in x and z directions respectively and n indicates time level t.

To apply the implicit Crank-Nicolson, first subdivide the continuum region of interest (x, z) into a set of rectangles of sides Δx and Δz, by equally spaced grid lines, parallel to z- axis, determined by $x_i = (i-1)\Delta x, i = 1, 2, 3, \dots, i_{max} + 1$ and parallel to x-axis, determined by $z_i = (j-1)\Delta z,\ j = 1, 2, 3, \dots, j_{max} + 1$. Then Eq. (4) at the grid points (i, j) and time step $n + \frac{1}{2}$ as the implicit Crank-Nicolson scheme (5) can be written,

$$\left[\frac{\partial C}{\partial t}\right]_{i,j}^{n+\frac{1}{2}} + \frac{1}{2}\left[(u + u_e)\frac{\partial C}{\partial x}\bigg|_{i,j}^{n} + (u + u_e)\frac{\partial C}{\partial x}\bigg|_{i,j}^{n+1}\right] + \frac{1}{2}\left[(w_e)\frac{\partial C}{\partial z}\bigg|_{i,j}^{n} + (w_e)\frac{\partial C}{\partial z}\bigg|_{i,j}^{n+1}\right] = \frac{1}{2}\left[\frac{\partial}{\partial z}K_z\frac{\partial C}{\partial z}\bigg|_{i,j}^{n} + \frac{\partial}{\partial z}K_z\frac{\partial C}{\partial z}\bigg|_{i,j}^{n+1}\right] - \frac{1}{2}(\lambda)(C_{i,j}^{n} + C_{i,j}^{n+1}$$
(6)

Making time dependent the subject of the equation (6)

$$\left[\frac{\partial C}{\partial t}\right]_{i,j}^{n+\frac{1}{2}} = -\frac{1}{2}\left[(u + u_e)\frac{\partial C}{\partial x}\bigg|_{i,j}^{n} + (u + u_e)\frac{\partial C}{\partial x}\bigg|_{i,j}^{n+1}\right] - \frac{1}{2}\left[(w_e)\frac{\partial C}{\partial z}\bigg|_{i,j}^{n} + (w_e)\frac{\partial C}{\partial z}\bigg|_{i,j}^{n+1}\right] + \frac{1}{2}\left[\frac{\partial}{\partial z}K_z\frac{\partial C}{\partial z}\bigg|_{i,j}^{n} + \frac{\partial}{\partial z}K_z\frac{\partial C}{\partial z}\bigg|_{i,j}^{n+1}\right] - \frac{1}{2}(\lambda)\left(C_{i,j}^{n} + C_{i,j}^{n+1}\right), \qquad (7)$$

where: $\frac{\partial C}{\partial t}\Big|_{i,j}^{n+\frac{1}{2}} = \frac{C_{i,j}^{n+1} - C_{i,j}^{n}}{\Delta t}$.

The main physical processes in the distribution of pollutants are advection and diffusion. In the mathematical model, the terms which are wind components (u, u_e) in it is the advection term and eddy diffusivity (K_z) in it is the diffusion term.

The backward difference is used to approximate the advection term,

$$(u + u_e)\frac{\partial C}{\partial x}\bigg|_{i,j}^{n} = \left(u_j + u_{e_{i,j}}\right)\left[\frac{C_{i,j}^{n} - C_{i-1,j}^{n}}{\Delta x}\right],$$
(8)

$$(u + u_e)\frac{\partial C}{\partial x}\bigg|_{i,j}^{n+1} = \left(u_j + u_{e_{i,j}}\right)\left[\frac{C_{i,j}^{n+1} - C_{i-1,j}^{n+1}}{\Delta x}\right],$$
(9)

$$(w_e)\frac{\partial C}{\partial z}\bigg|_{i,j}^{n} = w_{e_j}\left[\frac{C_{i,j}^{n} - C_{i-1,j}^{n}}{\Delta z}\right], \qquad (10)$$

$$(w_e)\frac{\partial C}{\partial z}\bigg|_{i,j}^{n+1} = w_{e_j}\left[\frac{C_{i,j}^{n+1} - C_{i-1,j}^{n+1}}{\Delta z}\right]. \qquad (11)$$

The central difference scheme is used to approximate the diffusion term,

$$\frac{\partial}{\partial z}\left(K_z\frac{\partial C}{\partial z}\right)\bigg|_{i,j}^{n} = \frac{K_z\frac{\partial C}{\partial z}\Big|_{i,j+\frac{1}{2}}^{n} - K_z\frac{\partial C}{\partial z}\Big|_{i,j-\frac{1}{2}}^{n}}{\Delta z}$$
$$= \frac{1}{\Delta z}\left(\frac{K_{j+1} + K_j}{2}\right)\left(\frac{C_{i,j+1}^{n} - C_{i,j}^{n}}{\Delta z}\right) - \frac{1}{\Delta z}\left(\frac{K_j + K_{j-1}}{2}\right)\left(\frac{C_{i,j}^{n} - C_{i,j-1}^{n}}{\Delta z}\right)$$
$$= \frac{1}{2(\Delta z)^2}\left[\left(K_{j+1} + K_j\right)C_{i,j+1}^{n} - \left(K_{j+1} + 2K_j + K_{j-1}\right)C_{i,j}^{n} +\right.$$

$$\left(K_j + K_{j-1}\right)C^{n}_{i,j-1}\Big]. \quad (12)$$

Similarly for,

$$\frac{\partial}{\partial z}\left(K_z\frac{\partial C}{\partial z}\right)\Bigg|^{n+1}_{i,j} = \frac{K_z\frac{\partial C}{\partial z}\Big|^{n+1}_{i,j+\frac{1}{2}} - K_z\frac{\partial C}{\partial z}\Big|^{n+1}_{i,j-\frac{1}{2}}}{\Delta z}$$
$$= \frac{1}{\Delta z}\left(\frac{K_{j+1}+K_j}{2}\right)\left(\frac{C^{n+1}_{i,j+1} - C^{n+1}_{i,j}}{\Delta z}\right) - \frac{1}{\Delta z}\left(\frac{K_j+K_{j-1}}{2}\right)\left(\frac{C^{n+1}_{i,j} - C^{n+1}_{i,j-1}}{\Delta z}\right)$$
$$= \frac{1}{2(\Delta z)^2}\left[\left(K_{j+1}+K_j\right)C^{n+1}_{i,j+1} - \left(K_{j+1}+2K_j+K_{j-1}\right)C^{n+1}_{i,j} + \left(K_j+K_{j-1}\right)C^{n+1}_{i,j-1}\right] \quad (13)$$

After approximating for all derivatives, equation (8) to (13) is substituted in to equation (7),

$$\frac{C^{n+1}_{i,j} - C^{n}_{i,j}}{\Delta t} = -\frac{1}{2}\left[\left(u_j + u_{e_{i,j}}\right)\left(\frac{C^{n}_{i,j} - C^{n}_{i-1,j}}{\Delta x}\right) + \left(u_j + u_{e_{i,j}}\right)\left(\frac{C^{n+1}_{i,j} - C^{n+1}_{i-1,j}}{\Delta x}\right)\right] - \frac{1}{2}\left[w_{e\,j}\left(\frac{C^{n}_{i,j} - C^{n}_{i,j-1}}{\Delta z}\right) + w_{e\,j}\left(\frac{C^{n+1}_{i,j} - C^{n+1}_{i,j-1}}{\Delta z}\right)\right] + \frac{1}{2}\left[\frac{1}{2(\Delta z)^2}\left[\left(K_{j+1}+K_j\right)C^{n}_{i,j+1} - \left(K_{j+1} + 2K_j + K_{j-1}\right)C^{n}_{i,j} + \left(K_j+K_{j-1}\right)C^{n}_{i,j-1}\right] + \frac{1}{2(\Delta z)^2}\left[\left(K_{j+1}+K_j\right)C^{n+1}_{i,j+1} - \left(K_{j+1}+2K_j + K_{j-1}\right)C^{n+1}_{i,j} + \left(K_j+K_{j-1}\right)C^{n+1}_{i,j-1}\right]\right] - \frac{1}{2}(\lambda)\left(C^{n}_{i,j} + C^{n+1}_{i,j}\right). \quad (14)$$

Time step $n+1$ are grouped on the left hand side whereas time step n on the right hand side to get the equation below,

$$C^{n+1}_{i,j} + \left(u_j + u_{e_{i,j}}\right)\frac{\Delta t}{2\Delta x}C^{n+1}_{i,j} - \left(u_j + u_{e_{i,j}}\right)\frac{\Delta t}{2\Delta x}C^{n+1}_{i-1,j} + w_{e\,j}\frac{\Delta t}{2\Delta z}C^{n+1}_{i,j} - w_{e\,j}\frac{\Delta t}{2\Delta z}C^{n+1}_{i,j-1} - \frac{\Delta t}{4(\Delta z)^2}\left(K_{j+1}+K_j\right)C^{n+1}_{i,j+1} + \frac{\Delta t}{4(\Delta z)^2}\left(K_{j+1}+2K_j+K_{j-1}\right)C^{n+1}_{i,j} - \frac{\Delta t}{4(\Delta z)^2}\left(K_j+K_{j-1}\right)C^{n+1}_{i,j-1} + \frac{\Delta t}{2}\lambda C^{n+1}_{i,j} = C^{n}_{i,j} - \left(u_j + u_{e_{i,j}}\right)\frac{\Delta t}{2\Delta x}C^{n}_{i,j} + \left(u_j + u_{e_{i,j}}\right)\frac{\Delta t}{2\Delta x}C^{n}_{i-1,j} - W_j\frac{\Delta t}{2\Delta z}C^{n}_{i,j} + W_j\frac{\Delta t}{2\Delta z}C^{n}_{i,j-1} + \frac{\Delta t}{4(\Delta z)^2}\left(K_{j+1}+K_j\right)C^{n}_{i,j+1} - \frac{\Delta t}{4(\Delta z)^2}\left(K_{j+1}+2K_j+K_{j-1}\right)C^{n}_{i,j} + \frac{\Delta t}{4(\Delta z)^2}\left(K_j+K_{j-1}\right)C^{n}_{i,j-1} - \frac{\Delta t}{2}\lambda C^{n}_{i,j} \quad (15)$$

with,

$$A_{i,j} = -\left(u_j + u_{e_{i,j}}\right)\frac{\Delta t}{2\Delta x}$$
$$B_j = -\left[\frac{\Delta t}{4(\Delta z)^2}\left(K_j + K_{j-1}\right) + w_{e\,j}\frac{\Delta t}{2\Delta z}\right]$$
$$D_{i,j} = 1 + \left(u_j + u_{e_{i,j}}\right)\frac{\Delta t}{2\Delta x} + w_{e\,j}\frac{\Delta t}{2\Delta z} + \frac{\Delta t}{4(\Delta z)^2}\left(K_{j+1} + 2K_j + K_{j-1}\right) + \frac{\Delta t}{2}(\lambda)$$
$$E_j = -\frac{\Delta t}{4(\Delta z)^2}\left(K_j + K_{j+1}\right)$$
$$F_{i,j} = \left(u_j + u_{e_{i,j}}\right)\frac{\Delta t}{2\Delta x}$$
$$G_j = \left[\frac{\Delta t}{4(\Delta z)^2}\left(K_j + K_{j-1}\right) + w_{e\,j}\frac{\Delta t}{2\Delta z}\right]$$
$$M_{i,j} = 1 - \left(u_j + u_{e_{i,j}}\right)\frac{\Delta t}{2\Delta x} - w_{e\,j}\frac{\Delta t}{2\Delta z} - \frac{\Delta t}{4(\Delta z)^2}\left(K_{j+1} + 2K_j + K_{j-1}\right) - \frac{\Delta t}{2}(\lambda)$$
$$N_j = \frac{\Delta t}{4(\Delta z)^2}\left(K_j + K_{j+1}\right)$$

the discretization form from equation (15) can be form for each $i = 2, 3, 4, \dots, i_{max} + 1$, $j = 2, 3, 4, \dots, j_{max}$ and $n = 1, 2, 3, \dots$, as:

$$A_{i,j}C^{n+1}_{i-1,j} + B_jC^{n+1}_{i,j-1} + D_{i,j}C^{n+1}_{i,j} + E_jC^{n+1}_{i,j+1} = F_{i,j}C^{n}_{i-1,j} + G_jC^{n}_{i,j-1} + M_{i,j}C^{n}_{i,j} + N_jC^{n}_{i,j+1}.$$

Equation (16) is appropriated for interior grid points and for the boundary grid points uses this boundary condition is used:

1. $C^{0}_{i,j} = 0$; for $i = 1, 2, \dots, i_{max}$ and $j = 1, 2, \dots, j_{max}$. (17)
2. $C^{n+1}_{1,j} = 0$; for $j = 1, 2, \dots, j_{max}$ and $n = 1, 2, 3, \dots$. (18)
3. $\left(1 + v_d\frac{\Delta z}{K_j}\right)C^{n+1}_{i,j} - C^{n+1}_{i,j+1} = \frac{Q\,\Delta z}{K_j}$; for $j = 1$, $i = 2, 3, 4, \dots, i_{max}$ and $n = 1, 2, 3,$ (19)

   where:
   v_d: the dry deposition velocity,
   Q : the emission rate of pollutant.
4. $C^{n+1}_{i,j_{max}+1} + C^{n+1}_{i,j_{max}} = 0$; for $i = 2, 3, 4, \dots, i_{max}$ and $n = 1, 2, 3, \dots$ (20)

The equations (16) to (20) have a tri-diagonal matrix structure and assure the existence of an inverse by solving it at each downwind step; the concentration values can be obtained at each vertical grid points from the downwind direction. It can be written as:

$$[H][C] = [U]$$

where H is the tri-diagonal matrix obtained from the coefficient of discretization form, C is the concentration of pollutants and U is the right hand side obtained from the coefficients from the discretization form.

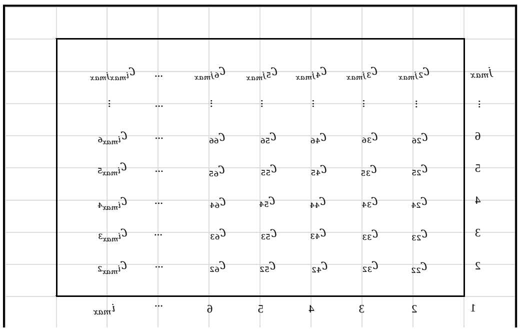

Figure 1. Discretization on grid.

Concentration corresponding to each position on the grid is showed and represented in figure 1. Concentration for time step $n+1$ is calculated or obtained from concentration values from previous time n, for example from the figure, a concentration value like C_{32} is obtained from concentration value C_{22} and so on.

The main goal of this thesis is to analyze the air pollution distribution using advection diffusion model which is solved by using the Crank-Nicolson method under the effect of eddy diffusivity and wind component such as large-scale wind and mesoscale wind. The distribution of pollution would be considered on different atmospheric conditions (unstable, neutral and stable) because in each of the atmospheric conditions, wind velocity and eddy diffusivity coefficient is different. These affects air pollution distribution thus whether it makes pollutants move far away from source or from ground level.

RESULT AND DISCUSSION

The Gaussian model as a boundary condition takes into account diffusive processes, advection with a mean air flow direction (large or mesoscale wind) and first order decay. The diffusive term here is used as an umbrella for various processes which have in common the tendency to lower concentration or temperature gradients.

Diffusion at the molecular scale can surely be neglected in the atmosphere, while variations and fluctuations at various scales within the velocity field are the causes for the observation of diffusion at a large scale. Moreover, turbulence adds another origin of diffusion.

Figure 2a through to 4b depicts the concentration on the ground level and also the concentrations along slices with constant x and varying y. In applying the boundary condition to our advection diffusion equation, the following assumptions were made.

1. The emission is continuous and constant
2. The wind speed (mesoscale wind) is constant in time and in elevation
3. In main wind direction, advection dominates over diffusion and dispersion.

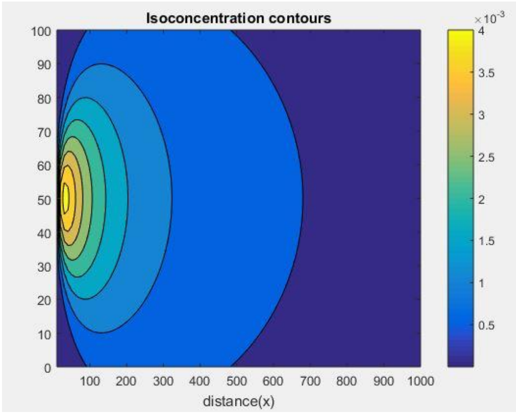

Figure 2a. Isoconcentration contours with $v = 0.05\ m/s$

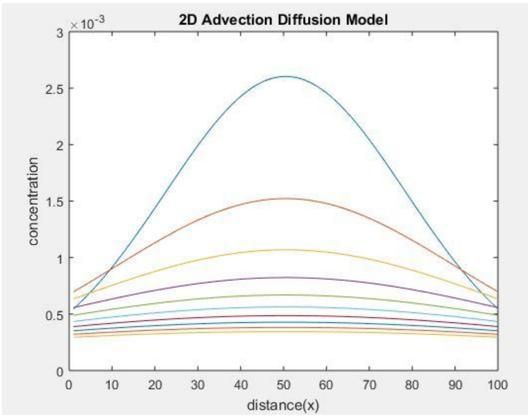

Figure 2b. The ground level concentration with $v = 0.05\ m/s$

Figure 2a shows the concentration on the ground with diffusivities of 0.2 and 1. The highest concentration on the ground can be observed approximately 50 m downstream with a velocity of 0.05 m/s. The concentration decrease slowly after a peak is reached at approximately 80 m. Figure 2b shows ground concentrations along slices with constant x and varying y. Clearly, all distributions are of Gaussian type and global peak concentration is found in the slice at $x = 600$. For $x < 600$, the shaped concentration have small standard deviation and local peak concentrations increase with x.

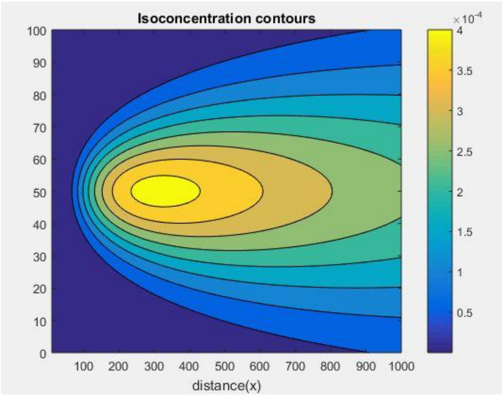

Figure 3a. Isoconcentration contours with $v = 0.5\ m/s$

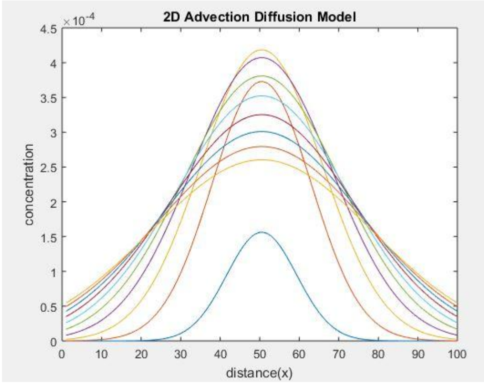

Figure 3b. The ground level concentration with $v = 0.5\ m/s$

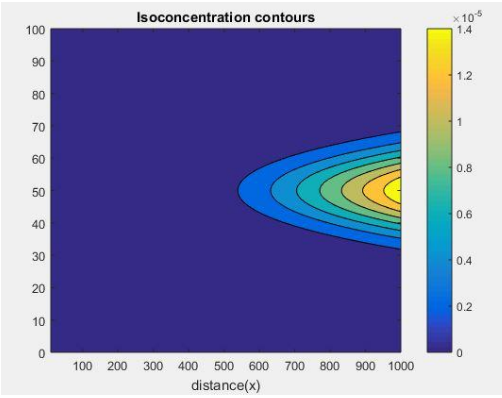

Figure 4a. Isoconcentration contours with $v = 5.0\ m/s$

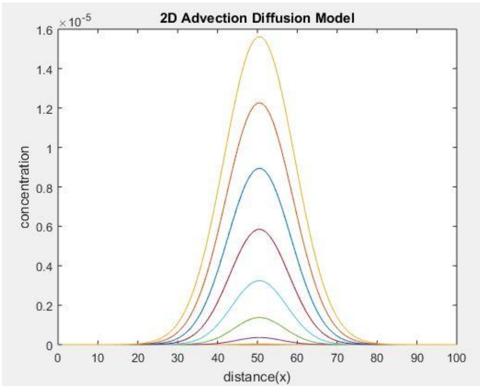

Figure 4b. The ground level concentration with $v = 5.0\ m/s$

In figure 3a, an increase in the velocity from $v = 0.05\ m/s$ to $v = 0.5\ m/s$ and the same diffusivities from figure 1a is shown. It could be seen that the highest concentration is at $300\ m$ downstream and decreases after the peak is reached approximately at $350\ m$. Figure 3b shows peak concentration at $x = 300$ with an increase in the velocity from $v = 0.005\ m/s$ to $v = 0.5 m/s$. For lower $x < 300$, the concentrations have small standard deviations and local peak concentrations increase with x. Beyond the slice with peak concentration, i.e for $x > 300$, the standard deviations increase and the local maximum decrease with x.

From figure 4a and 4b, the velocity is increased further from $v = 0.5\ m/s$ to $v = 5.0\ m/s$ and it could be observed that the concentration of the pollutants move farther away from source with the highest concentration at $950\ m$ downstream from the source. The peak concentration is seen in figure 4a at $x = 80$.

CONCLUSION

The distribution and movement of air pollutants in the atmosphere has been studied in urban area using advection-diffusion model with time dependent. The model was solved by numerical method using the implicit Crank-Nicolson finite difference scheme to discretize the area. For visualization purpose, the Gaussian model was used as a boundary condition to be able to plot an isoconcentration contour of the concentration of the pollutants in the region.

In reality, large scale wind and the mesoscale wind (the breeze that occur in a local area) have effect on air pollutants distribution. The large scale wind and the mesoscale wind transport the pollutants in the horizontal as well as in the vertical directions. With varying wind velocities, it is seen that the concentrations of pollutants move away or stay close to the source when the velocity of the wind is increased or decreased.

In considering ground level concentration in an unstable atmospheric condition, pollutants are carried away from the surface into the atmosphere as the surrounding air is warm. The advection is responsible for carrying the pollutants in the direction of the wind while the diffusion is responsible for carrying the concentration of the pollutants from a higher region to a lower region. As the air becomes warm, it is prone to further ascent and this phenomenon results in pollutants moving away from the source and receptors (building, water bodies, etc) which is scientifically good and healthy for living organism.

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REFERENCES

Agarwal, M. and Tandon, A. “Modeling of the urban heat island in the form of mesoscale wind and of its effect on air pollution dispersal”. *Applied Mathematical Modelling,* vol. 34, pp. 2520-2530, Dec. 2009.

Arystanbekova, N. Kh. “Application of Gaussian plume models for air pollution simulation at instantaneous emissions”. *Mathematics and Computers in Simulation*, vol. 67, pp. 451-458, Sept. 2004.

Awasthi, S., Khare, M. and Gargava, P. “General plume dispersion model (GPDM) for point source emission”. *Environmental Modeling and Assessment,* vol. 11, pp. 267-276, 2006.

Bell, J.N.B. and Treshow, M. *Air pollution and plant life, 2nd ed. John Wiley & Sons, Ltd.* West Sussex, England, 2002.

de Nevers, N. *Air Pollution Control Engineering*. Singapore: McGraw-Hill International Editions, 2000, pp. 119.

Dilley, J.F. and Yen, K.T. “Effect of a mesoscale type wind on the pollutant distribution from a line source”. *Atmospheric Environment,* vol. 5, pp. 843-851, April. 1971.

Enger, E. D., and Smith, B. F. *Environmental Science: A Study of Interrelationships (7th ed.).* Boston, Mass: McGraw-Hill, 2000.

Fenger, J. and Tjell, J.C. *Air Pollution from a Local to a Global Perspective.* Denmark: Polyteknisk Forlag, 2009, pp. 293.

Isikwue, B.C., Tsutsu, O., Utah, E.U. and Okeke, F.N. “Preliminary study on the estimation of horizontal dilution potential of air pollutants over some cities in Nigeria using wind data”. *Physical Review & Research International,* vol. 1(3), pp. 61-73, June. 2011.

J.S. Lin and L.M. Hildemann. “Analytical solutions of the atmospheric diffusion equation with multiple sources and height dependent wind speed and eddy diffusivities”. *Atmospheric Environment,* vol. 30, pp. 239-254, June. 1995.

Lakshminarayanachari. K., Sudheer, P. K. L., Siddalinga, P. M. and Pandurangappa, C. “Advection – diffusion numerical model of air pollutants emitted from an urban area source with removal mechanisms by considering point source on the boundary”. *International journal of Application or Innovation in Engineering and Management,* vol. 2, pp. 251-268, Feb. 2013.

Lakshminarayanachari. K., Suresha, C. M., Siddalinga, M. P. and Pandurangappa, C. “Numerical model of air pollutant emitted from an area source of primary and secondary pollutants with chemical reaction and gravitational settling with point source on the boundary". *International Journal of Research in Environmental Science and Technology*, vol. 3(1), pp. 9-18, Jan. 2013.

Lakzhminarayanachari. K., Sudheer, P. K. L., Siddalinga, P. M. and Pandurangappa, C. "A two dimensional numerical model of primary pollutant emitted from an urban area source with mesoscale wind, dry deposition and chemical reaction". *Atmospheric Pollution Research*, vol. 4, pp. 106-116, Nov. 2012.

Narayanachari, K.L., Suresha, C. M., Siddalinga, P. M. and Pandurangappa, C. "Time dependent advection diffusion model of air pollutants with removal mechanisms". *International Journal of Environmental Science*, vol. 5, pp. 34-50, July. 2014.

Pai, K. L. S., Laksminarayanachari, K. M., Siddalinga, P. and Pandurangappa, C. "Numerical model of air pollutant emitted from an area source of primary and secondary pollutants with chemical reaction and dry deposition". *International Journal of Science, Environment*, vol. 1, pp. 1-14, Nov. 2012.

Pandurangappa, C., Lakshminarayanachari, K. and Venkatachalappa, M. "Effect of mesoscale wind on the pollutant emitted from an area source of primary and secondary pollutants with gravitational settling velocity". *International Journal of Emerging Technology and Advanced Engineering*, vol. 2(9), pp. 325-334, Sept. 2012.

Sharan, M., Yadav. A. K., Singh, M. P., Agarwal, P. and Nigam, S. "A mathematical model for the dispersion of air pollutants in low wind conditions". *Center for Atmospheric Sciences*, vol. 30, pp. 1209-1220, Oct. 1995.

Suresha, C. M., Lakshminarayanachari, K., Prasad, M. S. and Pandurangappa, C. "Numerical model of air pollutant emitted from an area source of primary pollutant with chemical reaction". *International Journal of Application or Innovation in Engineering and Management*, vol. 2(3), pp. 132 – 144, March. 2013.

Venkatachalappa, M., Khan, S. K. and Kakamari, K. A. G. "Time dependent mathematical model of air pollution due to area source with variable wind velocity and eddy diffusivity and chemical reaction", in *Proc. Indian Natn Sci Acad 69, A, No. 6*, 2003, pp. 745-758.

World Meteorological Organization. *WMO/IGAC Impacts of megacities on air pollution and climate.* Geneva2, Switzerland: World Meteorological Organization, 2012, pp. 22.