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LINGUISTIC LABELS BASED METHODOLOGY FOR FUZZY GROUP DECISION MAKING

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ABSTRACT

In this paper, we review our research works on gradual improvements to fuzzy group decision-making methodologies. Firstly, a single-level semi-numeric method, i.e., linguistic labels representation with fuzzy sets computation, and a fully non-numeric method, i.e., linguistic labels representation with labels manipulation for solving pairwise fuzzy group decision making problems are discussed. Secondly, the extension of these methods into a hierarchical semi-numeric and a hierarchical non-numeric methods are described. Finally, the methods are verified and applied to solving some real cases of fuzzy group decision-making problems such as a waste disposal methods selection problems.

Keywords : group decision making, fuzzy logic, linguistic label, preference modeling, multiple criteria hierarchical analysis

Our research works discussed here are devoted to gradually revising and improving the previously developed fuzzy group decision-making methodologies. For this purpose, linguistic labels based methodologies which are more flexible and suitable to be applied to solving real cases of pairwise fuzzy group decision-making problems are developed.

We propose a semi-numeric method [1], i.e., linguistic labels representations with fuzzy sets computation, and a fully non-numeric method [2], i.e., linguistic labels representations with non-numeric computation for pairwise fuzzy group decision making. These methods can be used to identify the preferred alternatives. Satisfaction degree to each alternative preference is accommodated into the proposed methods [3]. These methods deal with single-level structure problems.

For dealing with a large and a more complex problem, the semi-numeric and non-numeric methods are extended into a hierarchical semi-numeric method [4], [5] and a hierarchical non-numeric method [6], respectively. These methods are able to solve multiple criteria hierarchical structure problems. The methods are verified and applied to solving some cases of fuzzy group decision making problems, e.g., advertising media selection, a waste disposal method selection, and agricultural based industries project selection problems. The hierarchical semi-numeric method is compared to the Saaty's method of AHP [7]. The possibility to improve the methods are also discussed.

Fuzzy Preference Relations

In the numeric method for pairwise fuzzy group decision making [8], an element of a fuzzy preference relation matrix R^k on a set of alternatives S, for individual decision maker k is expressed by a membership function $\mu_R k\,(S_i, Sj) : S \times S \rightarrow [0,1]$ as follows:

$$\mu_{R^k}(s_i,s_j)=\begin{cases}1 & \text{if } s_i \text{ is definitely preferred to } s_j,\\ c\in(0.5,1) & \text{if } s_i \text{ is slightly preferred to } s_j,\\ 0.5 & \text{if } s_i \text{ is indifference to } s_j,\\ d\in(0,0.5) & \text{if } s_j \text{ is slightly preffered to } s_i,\\ 0 & \text{if } s_j \text{ is definitely preffered to } s_i.\end{cases} \quad (1)$$

This direct single-point numerical representation is impractical to be implemented, because the decision-makers tend to give linguistically more natural evaluation values. Moreover, the computation models are rigid and some information may be lost in the computation process. The numeric single-point representation and computation are extended in [1] by using linguistic labels representation and with fuzzy sets computation. The decision-makers may express their fuzzy preference relations in 13 labels according to the following equation:

$$R^k(s_i,s_j)=\begin{cases}DP & \text{if } s_i \text{ is preferred to } s_j \text{ in definite degree},\\ VHP & \text{if } s_i \text{ is preferred to } s_j \text{ in very high degree},\\ HP & \text{if } s_i \text{ is preferred to } s_j \text{ in high degree},\\ MP & \text{if } s_i \text{ is preferred to } s_j \text{ in moderate degree},\\ LP & \text{if } s_i \text{ is preferred to } s_j \text{ in low degree},\\ VLP & \text{if } s_i \text{ is preferred to } s_j \text{ in very low degree},\\ AS & \text{if } s_i \text{ about the same as } s_j,\\ VLD & \text{if } s_j \text{ is preferred to } s_i \text{ in very low degree},\\ LD & \text{if } s_j \text{ is preferred to } s_i \text{ in low degree},\\ MD & \text{if } s_j \text{ is preferred to } s_i \text{ in moderate degree},\\ HD & \text{if } s_j \text{ is preferred to } s_i \text{ in high degree},\\ VHD & \text{if } s_j \text{ is preferred to } s_i \text{ in very high degree},\\ DD & \text{if } s_j \text{ is preferred to } s_i \text{ in definite degree}.\end{cases} \quad (2)$$

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The preference values are reflective-reciprocal with AS as the center. He labels are converted into and processed in partially ordered triangular fuzzy numbers (TFN). The computation models use direct and indirect approaches. However, they are different with the numeric single-point computation models [8] in the following ways: (1) the core concept is extended into a fuzzy core concept which allows fuzzy member of alternatives in a solution set; (2) thresholds are eliminated from the computation models; (3) *neat*-OWA (ordered weighted average) aggregation operators [9] are used both for aggregating the individual preferences into a group preference and for representing the aggregation guided by linguistic quantifier such as *most*; and (4) a set of criteria which may have the same or different weight are considered explicitly. The computation produces solutions in TFN that can be converted back into the corresponding labels by using a similarity method [1].

The conversion process may be time consuming and it may decrease the results' accuracy. To avoid these drawbacks a fully non-numeric method is proposed in [2]. The computation models are simplified, i.e., the fully ordered labels are directly processed non-numerically, and OWA operators for a non-numeric ordinal environment [10] are used on some of the processes.

The non-numeric computation models also use direct and indirect approaches. However, the computation procedures are simplified, i.e., there is no transformation process from labels into TFN values and vice versa. The computation produces at most one preferred alternative in a label form. The non-numeric method is especially suitable for a group decision making in which a full consensus is the role for selecting the alternatives.

We also propose to use the linguistic labels for expressing the corresponding degrees of satisfaction to the preference values [3]. The degree of satisfaction is represented by using five-points labels as stated in Eqn. (3). So, there will be preference matrices R and their corresponding satisfaction matrices T. T are symmetrical matrices, i.e., t_{ij}^k is equal to t_{ji}^k. For example, $r_{ij}^k = HP$ and $t_{ij}^k = VH$ which mean that by the decision maker k the satisfaction degree of the alternative i is *preferred* to the alternative j in *high degree is very high.*

$$t_{ij}^k = \begin{cases} \text{VH} & \text{if the satisfactin degree to the } r_{ij}^k \text{ is veryhigh,} \\ \text{HI} & \text{if the satisfactin degree to the } r_{ij}^k \text{ is high,} \\ \text{ME} & \text{if the satisfactin degree to the } r_{ij}^k \text{ is medium,} \\ \text{LO} & \text{if the satisfactin degree to the } r_{ij}^k \text{ is low,} \\ \text{VL} & \text{if the satisfactin degree to the } r_{ij}^k \text{ is verylow.} \end{cases} \quad (3)$$

The satisfaction matrices can be processed by using the semi-numeric method.

Hierarchical Analysis

We extend the semi-numeric and the fully non-numeric methods for solving multiple criteria hierarchical structure problems which are commonly faced in the real applications. Each problem is tailored into a focus, criteria and alternatives. The structuring process is similar to that in the AHP (Analytic Hierarchy Process) [7].

The evaluations are done in the criteria level and than in the alternatives level on each criterion. For example the waste disposal method selection problem can be tailored in a hierarchical structure as shown in Fig.1. level one is the one focus of the analysis, level two consist of criteria used for selecting the alternatives, and level three is the considered alternatives. For evaluation on the criteria level, the term "is preferred to" of Eqn. (2) is replaced by "is more important than".

In the hierarchical semi-numeric method [4], [5], the linguistic labels are converted into triangular fuzzy numbers (TFN) as shown in Fig.2 (a) and then processed by using direct and indirect approaches[1]. The computation models are summarized in Table 1. The computation result fuzzy relative weights of the criteria in TFN form. Evaluation on the alternative level, which is the same as that on the single-level structure, results the preferred alternatives on each criterion.

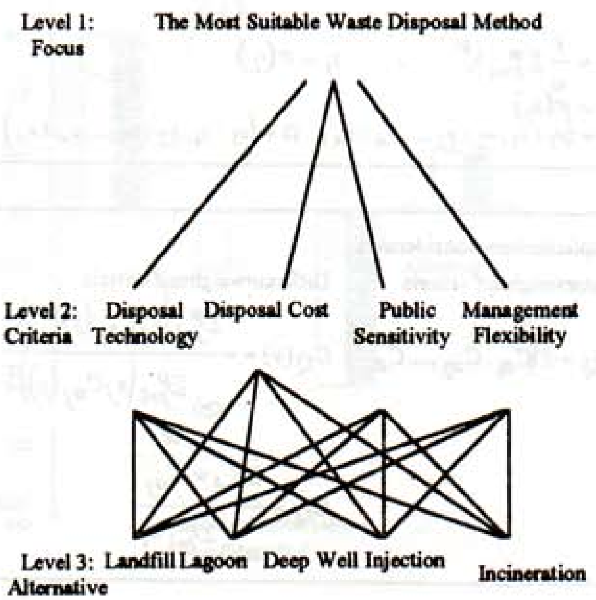
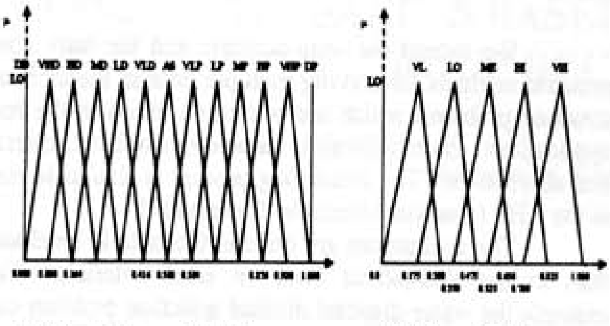


Figure 1. A hierarchical structure of the waste disposal method selection problem

The fuzzy relative weights of the criteria are used in the fuzzy weighted average or the special case of *neat*-OWA operator for aggregating solutions on each criterion into solutions on overall criteria.



(a) Preference values represented by TFN. (b) Weight or satisfaction degree represented by TFN.

Figure 2. The mapping of linguistic labels into the corresponding TFN.

Table 1. Formulas of the semi-numeric method for solving a pairwise fuzzy group decision making problem

Formula for each criterion

Direct approach

$$h_i^k = \frac{1}{n-1}\Sigma_{j=1, j\neq i}^{n} r_{ij}^k$$

$$h_i = \frac{1}{m}\Sigma_{k=1}^{m} h_i^k$$

$$v_i = F(h_i)$$

$$C = \{v_1/s_1, v_2/s_2, \ldots, v_n/s_n\}$$

Indirect approach

$$r_{ij} = \begin{cases} F\left(r_{ij}^k\right) & \text{if } i \neq j, \\ 0 & \text{otherwise} \end{cases}$$

$$F\left(r_{ij}\right) = \frac{\Sigma_k {r_{ij}^k}^{\alpha+1}}{\Sigma_k {r_{ij}^k}^{\alpha}}, \quad \alpha \geq 0,$$

$$z_i = F(r_{i.})$$

$$\Omega = \{z_1/s_1, z_2/s_2, \ldots, z_n/s_n\}$$

Explicitcriteriaconsideraion

Sameweightsof criteria

$$C_Q = F\left(C_{a_1}, C_{a_2}, \ldots, C_{a_p}\right)$$

Differentweightsof criteria

$$C_Q(y) = \frac{\Sigma_{j=1}^{p}\left(x_j C_{a_j}(y)\right)^{\alpha+1}}{\Sigma_{j=1}^{p}\left(x_j C_{a_j}(y)\right)^{\alpha}} \quad \alpha \geq 0$$

or

$$C_Q = \frac{\Sigma_{j=1}^{p} w_i C_{a_j}}{\Sigma_{j=1}^{p} w_i} \quad \alpha \geq 0$$

Legend :

h_i^k : The degree to which individual k supports alternative s_i

h_i : The degree to which all indivudual supports s_i

v_i : most individual supports s_i

C : The solution set on each criterion

r_{ij} : The element of a group fuzzy - preference matrix

z_i : similiar to v_i

Ω : similiar to C

C_Q : The solution on all criteria

$F(.)$: The neat OWA aggregation operator

In the hierarchical non-numeric method [6], on the criteria level, each decision-maker expresses his/her opinion about the importance/weight of criterion towards the focus of the analysis independently. The possible values of the weight are *very high, high, medium, low and very low*. Evaluations of all the decision-makers are aggregated to obtain the final weight of each criterion by using formulas explained in [2], [10].

Preferences to the alternatives of all decision-makers are processed by using the direct and indirect approaches of the non-numeric formulas summarized in Table 2. These formulas result solution on each criterion which can be aggregated into the final solution with respect to the weight of criteria obtained previously.

Verification and Discussion

Let us consider a 4 criteria and 4 alternatives fuzzy group decision-making case shown in Fig. 1. In this case there are 4 decision-makers, namely: (1) an environmental division manager, (2) an environmental bureaucrat from a municipal city, (3) an environmental expert from a university, and (4) and environmental expert from an environmental engineering consulting company. The evaluations are done directly on the alternative level under each criterion.

Table 2. Formulas of the non-numeric method for solving a pairwise fuzzy group decision making problem

Formulafor eachcriterion

Direct approach

$$h_i^k = \wedge_{j=1, j\neq i}^{n} r_{ij}^k$$

$$h_i = \wedge_{k=1}^{m} h_i^k$$

$$v_i = \begin{cases} v_i(w_j \wedge b_j) & \text{if } \forall h_i \geq AS, \\ \wedge_{k=1}^{m} h_i^k & \text{otherwise} \end{cases}$$

$$C = \{v_1/s_1, v_2/s_2, \ldots, v_n/s_n\}$$

Indirectapproach

$$z_i = \begin{cases} v_i(w_j \wedge b_j) & \text{if } \forall r_i \geq AS, \\ \wedge_{j=1, j\neq i}^{n}(r_{ij}) & \text{otherwise} \end{cases}$$

$$\Omega = \{z_1/s_1, z_2/s_2, \ldots, z_n/s_n\}$$

Explicit criteria considerаtion

Same weightsof criteria

$$C_Q = \begin{cases} v_i(b_j \wedge w_j) & \text{if } \forall C_{ai} \geq AS, \\ \wedge_{i=1}^{l} C_{ai} & \text{otherwise} \end{cases}$$

Differentweights of criteria

$$C_Q = \begin{cases} \wedge_{i=1}^{l}[\neg(t_i) \wedge C_{ai}] & \text{if } \forall C_{ai} \geq AS, \\ \wedge_{i=1}^{l} C_{ai} & \text{otherwise} \end{cases}$$

Legend :

h_i^k : The degree to which individual k supports alternativ e s_i

h_i : The degree to which all indivudual supports s_i

v_i : most individual supports s_i

C : The solution set on each criterion

r_{ij} : The element of agroup fuzzy - preference matrix

z_i : similiar t o v_i

Ω : similiar t o C

C_Q : The solution on all criteri

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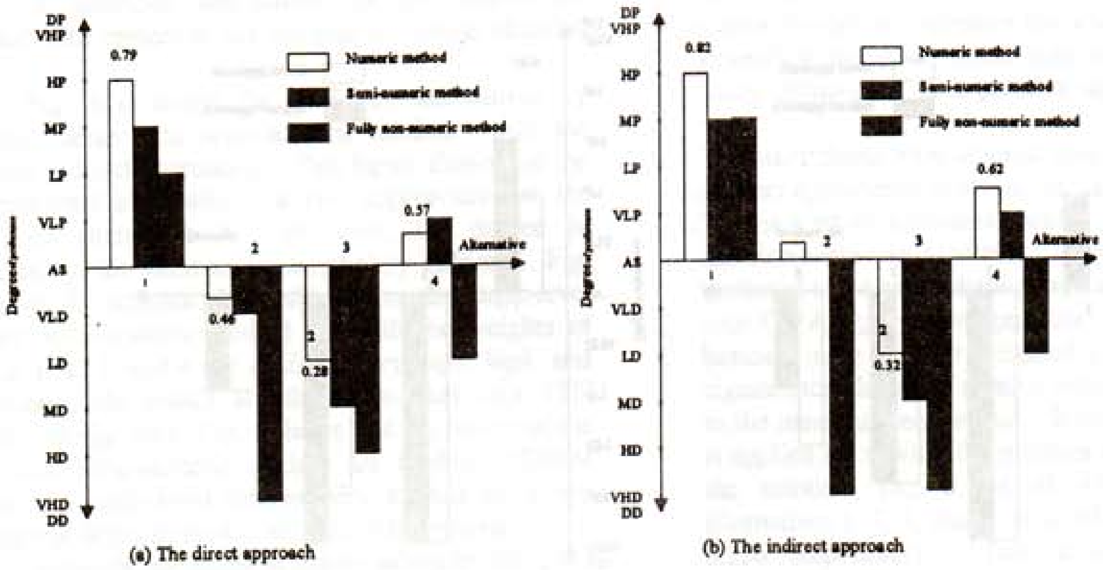

Figure 3. Solution of a waste disposal method selection by single-level structure methods

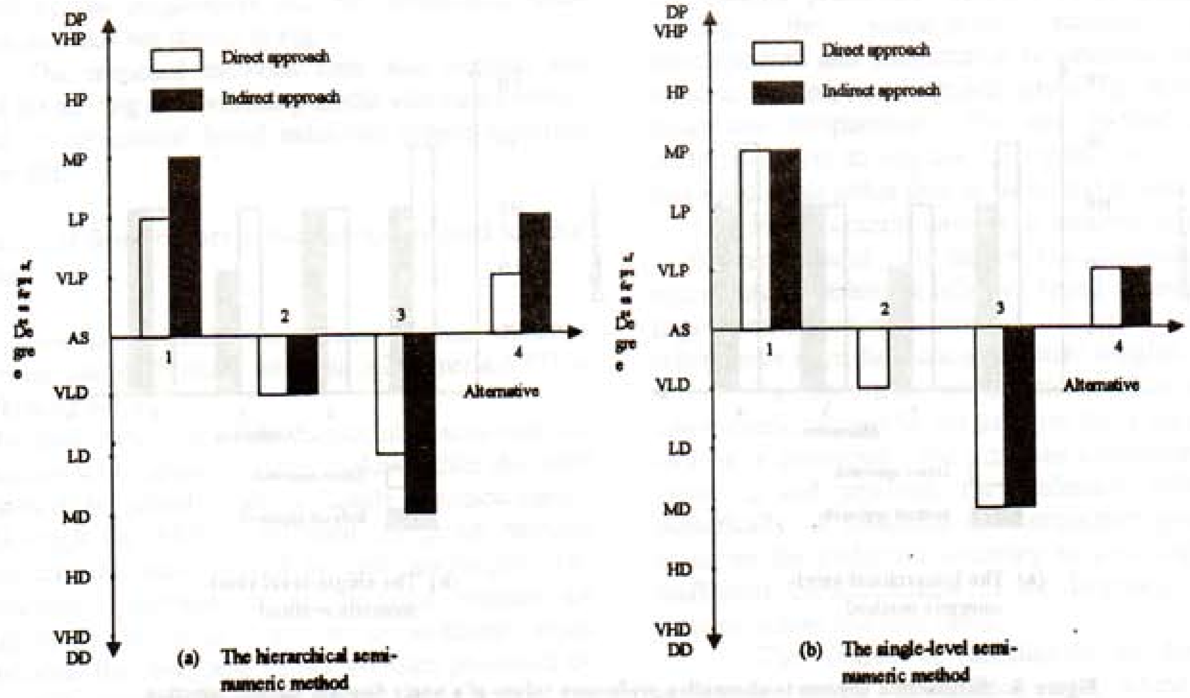

Figure 4. Solution of a waste disposal method selection by using the hierarchical and single-level semi-numeric method

Fig. 3 shows solutions of the single-level numeric, semi-numeric, and fully non-numeric methods each with the direct and indirect approaches. It shows that the most preferred alternatives on all methods and on all approaches are the same, i.e., alternative 1. However, the degree of preferences to the alternatives are slightly different.

The Fully non-numeric method gives only one preferred alternative. The numeric and the semi-numeric methods are suitable for full and partial consensus, while the fully non-numeric method is suitable especially for full consensus cases. Full consensus means that an alternative can be considered as the solution, if all the decision-makers have no objection to that alternative. Partial consensus means that an alternative is still possible as the solution, even if one or more decision maker(s) dis-prefer(s) to that alternative but it is preferred by the group of decision-makers as a whole.

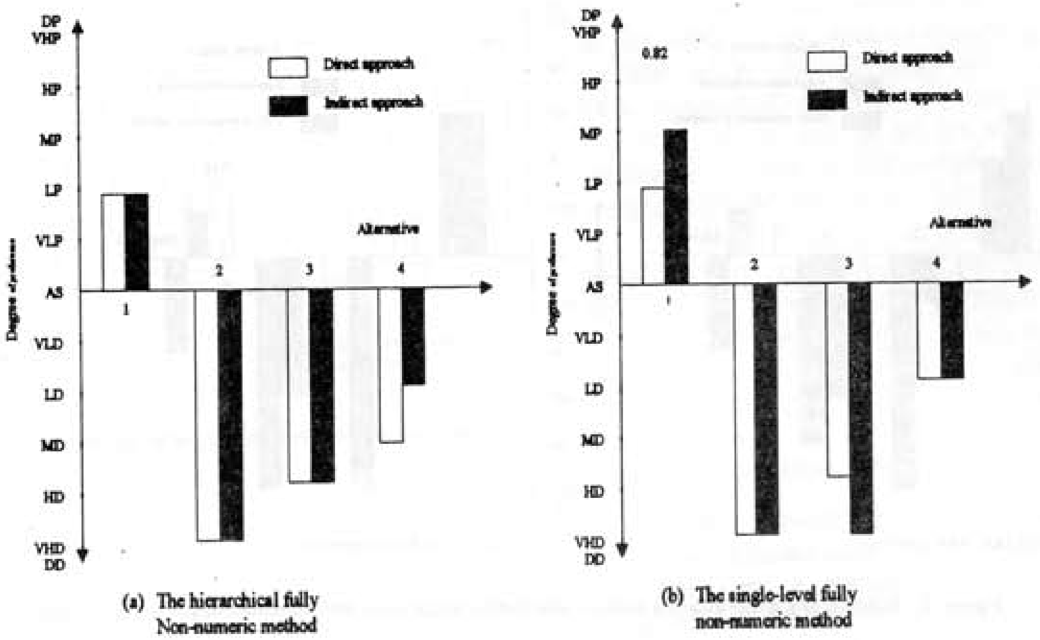

Figure 5. Solutions of a waste disposal method selection by usingthe hierarchical and single-level fully non-numeric methods

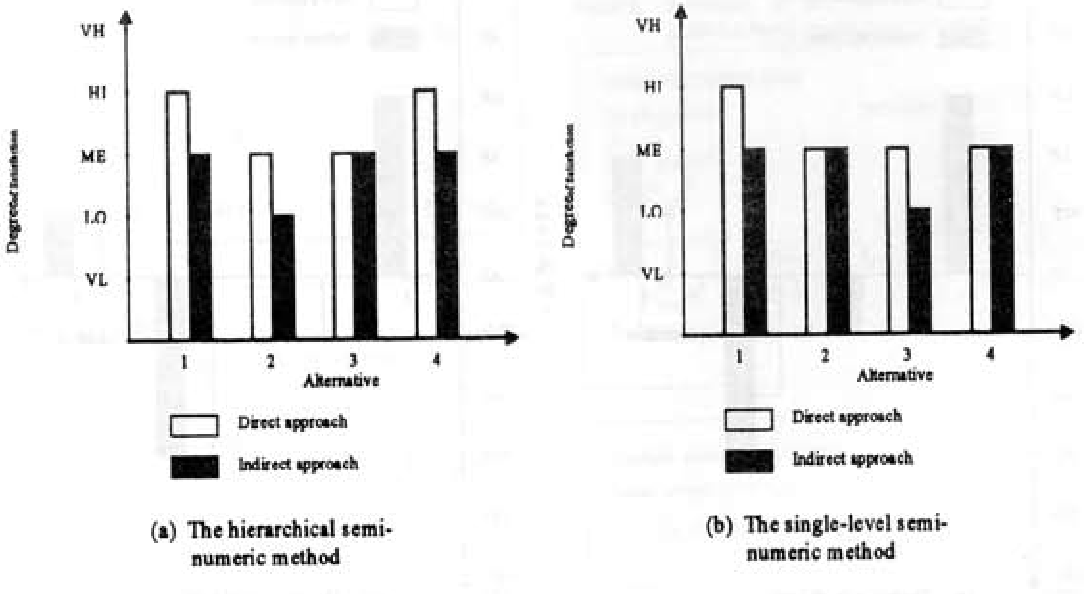

Figure 6. Satisfaction degrees to alternative preference values of a waste disposal method selection

The waste disposal selection problem is then solved by using the hierarchical semi-numeric method. Firstly, the decision-makers assign preference relation on the criteria level. Secondly, the decision-makers assign preference relation to each pair of alternatives on each criterion.

The data obtained from the first evaluation are processed to identify the weight of criteria. These can be done by using formulas listed on Table 1. The result is a set of relative weights of the criteria in TFN. In our case, the weights of criterion 1, 2, 3 and 4 are (0.316, 0.499, 0.768), (0.525, 0.717, 0.967), (0.361, 0.546, 0.808), and (0.095, 0.232, 0.584), respectively.

The data obtained from the second evaluation are processed to identify the preferred alternatives on each criterion. Then, by using the fuzzy weighted average model the preferred alternatives on all criteria are identified with respect to the weights of criteria obtained above.

Fig. 4(a) shows the preferred alternatives by using the hierarchical semi-numeric method with the direct and indirect approaches. This figure shows that the most preferred alternatives on two approaches are the same, i.e., alternative 1. However, the degree of preferences to the alternatives are slightly different. Fig. 4(b) shows the solution obtained by using the single-level structure semi-numeric method [1] with the weights of criterion 1, 2, 3, and 4 are *medium, very high, high,* and *low*, respectively which are then converted into TFN according to Fig. 1(b). Fig. 4 shows that the results of the hierarchical semi-numeric method are slightly different with that the single-level semi-numeric method due to the difference in determining the weights of the criteria.

Similarly, the problem can be solved by using the hierarchical fully non-numeric method. Because the non-numeric method follows full consensus rule, it suggests only one alternative, that is alternative 1 (Fig. 5). In term of most preferred alternatives, all methods suggest that alternative 1 is the best.

The satisfaction degree to each preference values obtained by the single-level and the hierarchical semi-numeric methods are shown in Fig. 6.

The proposed methods were also verified and applied for solving an advertising media selection problem [1] and an agricultural based industries project selection problem [2].

Hierarchical Semi-numeric Method Compared to AHP Method

The hierarchical semi-numeric method has some similarities and differences with the AHP method [7] in the following ways:

- The main intention of the hierarchical semi-numeric method is for group decision-makers while the AHP method is actually for a single decision-maker. Although the AHP is extended for group decision makers, the main model is still unchanged, i.e., preference matrices of all decision makers are aggregated by using geometric or arithmetic mean operator, the composite values are then processed by using the ordinary AHP.
- In both of the two methods let each decision-maker evaluates each pair of options. In the hierarchical semi-numeric method the evaluations are expressed in two groups of seven labels, see Eqn. (2), which are then converted into and processed in TFN. In the AHP method the evaluations are expressed in a group of nine ordinal values from 1 (equal importance) to 9 (one option absolutely more important than the other). This judgement is r_{ij}^{a}, the (i, j)th entry of the ath criterion matrix. r_{ji}^{a} is taken to be $1/r_{ij}^{a}\ \forall_{ij}$ and $r_{ii}^{a} = 1\forall_{i}$. It is possible to extend the ordinal evaluation values of AHP into fuzzy evaluation values, e.g., the evaluation can be expressed into about 1, about 3, etc. Extending the AHP model into a group-AHP or a fuzzy group-AHP is a possible topic for the future work.
- The hierarchical semi-numeric method uses direct and indirect approaches to derive the intended solution set. That is a set of alternatives which is preferred by the decision-maker as a whole. There will be a set of preferred and a set of dispreferred alternatives. It *uses neat* OWA aggregation operators. The AHP method basically uses a linear weighted formula for finding eigenvector, the set of relative weights, corresponding to the maximum eigenvalue. When the AHP method is applied for solving the problem described in Fig.1, the solution, i.e., a set of relative weights of alternatives 1, 2, 3, and 4, is 0.503, 0.149, 0.092, and 0.256 respectively. This shows that the best alternative is alternative 1, which is also suggested by the semi-numeric method.

CONCLUSIONS

For improving the flexibility and practicality of the numeric preference relations based group decision making, the single-point numeric preference representation and computation is extended into a semi-numeric method, i.e., linguistic labels representation with fuzzy sets computation. The new method allows the decision-makers to express their preference relations in linguistic labels rather than in numerical values.

Fuzzy criteria have been incorporated explicitly to the new model. It allows the decision-makers to express their preference relations based on each criterion. It gives results on each criterion and on all criteria. The criteria may have the same or different weights.

To improve the computational efficiency, a fully non-numeric method for the pairwise fuzzy group-decision making is developed. The proposed non-numeric method expresses and processes the preference relations non-numerically. It simplifies the computation processes and preserves the preference accuracy by avoiding the use of inefficient transformations of the linguistic labels into numeric values and vice versa.

The degree of satisfaction to the assigned preference values is defined and added into the representation and computation models. This allows the decision-makers to express the corresponding satisfaction degree of the preference values.

The single level semi-numeric and non-numeric methods are extended for solving multiple criteria hierarchical structure pairwise fuzzy group decision-making problems. By doing so, the proposed methods are able to solve more complex and larger group decision-making problems.

All of the proposed methods have been verified, compared and applied to solving some real cases, e.g., an advertising media selection, an agroindustrial project selection and a waste disposal method selection. Moreover, the hierarchical semi-numeric method is compared to the AHP method.

Degree of satisfaction and the preference values are identified and analyzed separately. Other representation method, e.g., a fuzzy set of type 2 can be investigated for representing and processing the preference values together with the corresponding satisfaction degrees simultaneously.

The semi-numeric method, non-numeric method and their variations which have been developed in this research can be used as one of the possible basis for developing a group decision support system.

The consistency of the preference matrices are computed by using a similar method to that in AHP. In this case, the labels are transformed to TFN and the analysis considers the middle values of the corresponding TFN. Investigation of developing a model to determine the consistency directly from matrix of labels can be considered for the future work.

The hierarchical semi-numeric method has similarities and differences with the AHP method. For future work, it can be investigated the possibility and then extend the hierarchical semi-numeric method into a network semi-numeric method which is similar to the analytic network process [11].

REFERENCES

Marimin; M. Umano, I. Hatono & H. Tamura. 1998. Linguistic labels for expressing fuzzy preference relations in fuzzy group decision making", IEEE Trans. On Systems, Man and Cybernetics 28(1).

Marimin; M. Umano, I. Hatono & H. Tamura. 1997. Non-numeric method for pairwise fuzzy group decision analysis, J. Intell. Fuzzy Systems 5 (4).

Marimin; M. Umano, I. Hatono & H. Tamura. 1996. Non-numeric preference relations and their satisfaction degrees for pairwise fuzzy group-decision making. Proc. 35th IEEE Conf. on Decision and Control, Kobe, Japan, pp. 108-109.

Marimin; M. Umano, I. Hatono & H. Tamura. 1997. Multiple criteria multi-level semi-numeric method for pairwise fuzzy group-decision making, Proc. Int. Conf. on Methods and Applications of Multicriteria Decision Making, Mons, Belgium, pp. 30 – 33.

Marimin; M. Umano, I. Hatono & H. Tamura, Hierarchical semi-numeric method for pairwise fuzzy group decision making. IEEE Trans. On systems, Man and Cybernetics, (submitted).

Marimin; M. Umano, I. Hatono, S. Tomiyama & H. Tamura. 1997. Towards hierarchical analysis of non-numeric method for pairwise fuzzy group-decision making. Proc. 2nd Asian Control Conf., Seoul, Korea, July 22-25, pp. 567 – 570.

Saaty,T.T. 1980. The analytic hierarchy process. McGraw-Hill, New York.

Saaty, T.T. 1996. Decision making with dependence and feedback: The analytic network process. RWS Publications, Pittsburg

Kacprzyk, J; M. Fedrizzi & H. Nurmi. 1992. Group decision making and consensus under fuzzy preferences and fuzzy majority. Fuzzy Sets and Systems 49 (1): 21 – 31.

Yager, R.R. 1993. Families of OWA operators. Fuzzy Sets and Systems 59:125 – 148.

Yager, R.R. 1993. Non-numeric multi-criteria multi-person decision making. Group Decision and Negotiation 2(1): 81 – 93.